# TWO-PHASE COUNTERCURRENT FLOW THROUGH A BED OF PACKING. VII.* 

# PRESSURE DROP AT FLOODING AND CRITICAL FLOW RATES OF BOTH PHASES IN PACKED BED OF SPHERES 

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A relation was obtained which on a high statistical level makes possible the calculation of the friction factor for the gaseous phase at flooding for different dimensions of particles of the packing and different physical properties of liquids employed. In this work is demonstrated how the pressure drop at flooding may be used for determination of pressure drop at different conditions and relations are derived for calculation of critical flow rates of individual phases.

The friction factor for the gaseous phase at countercurrent two-phase flow through a packed bed was defined by relation ${ }^{1}$

$$
\begin{equation*}
\Delta P_{g}=\psi_{\mathrm{g}}\left(l / d_{\mathrm{c}}\right)\left\{G_{\mathrm{g}}^{2} /\left[2 \varrho_{\mathrm{g}}(e-z)^{3}\right]\right\} . \tag{1}
\end{equation*}
$$

If values of friction factor defined in this way are plotted as a function of Reynolds number in logarithmic coordinates, a system of lines is obtained with the liquid flow rate as a parameter. The shape of the dependence is obvious from Fig. 1 and Figs 1 to 5 of the preceding paper ${ }^{2}$. At low liquid flow rates the course of the dependence of friction factor on Reynolds number for gas is close to that one for dry packed bed. At the increasing liquid flow rate, the dependence is deformed in a different manner and becomes straight at high liquid flow rates.

Though the dependences for different liquid flow rates are mutually quite different, it is obvious from all the given figures that values of the friction factor at flooding are situated for the given packing and physical properties of the system in given coordinates on the same straight line.

As is obvious from Fig. 2, these straight lines have for different dimensions of particles of the bed the same slope and are mutually only shifted.

For the friction factor at flooding is thus valid the relation

[^0]\[

$$
\begin{equation*}
\psi_{\mathrm{gz}}=A \mathrm{Re}_{\mathrm{gz}}^{\mathrm{B}} . \tag{2}
\end{equation*}
$$

\]

The curve analytically expressed by Eq. (2) is crossing the curve of dependence of friction factor on Reynolds number for the value of parameter $G_{1}=0$ in the point for which the corresponding gas flow rate $G_{\mathrm{gz}}$ may be determined from the earlier derived ${ }^{3}$ relation for flooding given by equation

$$
\begin{equation*}
\left[\left(\frac{G_{\mathrm{gz}}}{G_{1}}\right)^{2} \frac{\varrho_{\mathrm{I}}}{\varrho_{\mathrm{g}}}\right]^{0.25}=0.73\left(\frac{2 g e^{3} \varrho_{1}^{2} d_{\mathrm{c}}}{G_{1}^{2}}\right)^{0.25}-1.2\left(\frac{\mu_{\mathrm{l}}}{\mu_{\mathrm{w}}} \frac{\varrho_{\mathrm{w}}}{\varrho_{\mathrm{t}}}\right)^{0.265}\left(\frac{d_{\mathrm{e}_{\mathrm{o}}}}{d_{\mathrm{e}}}\right)^{0.23} \tag{3}
\end{equation*}
$$

from which by substituting for $G_{1}=0$ we obtain

$$
\begin{equation*}
G_{\mathrm{gz}}\left(G_{1}=0\right)=0.533\left(2 g e^{3} \varrho_{1} \varrho_{\mathrm{gz}} d_{\mathrm{e}}\right)^{1 / 2} . \tag{4}
\end{equation*}
$$



Fig. 1
Dependence of $\psi_{\mathrm{g}}$ on $\mathrm{Re}_{\mathrm{g}}$ for the Water-Air System, Packing of Spheres of 10 mm Diameter
$G_{1}$ in $\mathrm{kg} \mathrm{m}^{-2} \mathrm{~s}^{-1} .1$ Dry packing; 2 wetted packing, $G_{\mathrm{i}}=0, z=0.03 ; 2^{\prime}$ wetted packing at flooding $G_{\mathrm{I}}=0$ (extrapolated dependence); $3 G_{1}=0.208 ; 40.306 ; 50.833 ; 61.42$; $72.55 ; 83.30 ; 94 \cdot 12 ; 105.50 ; 116.87 ; 12$ 7.54; 13 8.25; 14 9.62; 15 11.0; 16 13.7; 17 16.5; 18 19.9.


Fig. 2
Dependence of $\psi_{\mathrm{g} 5}$ and $\psi_{\mathrm{gz}}$ on $\mathrm{Re}_{\mathrm{g}}$ for Packing of Spheres of 10,15 and 20 mm Diameter for Water-Air System

| Diameter, mm | Water | Dry packing |
| :---: | :---: | :---: |
| 10 | $\bullet$ | $\oplus$ |
| 15 | $\circ$ | $\ominus$ |
| 20 | $\odot$ | $\odot$ |

Relation (2) may be also written in the form

$$
\begin{equation*}
\frac{\psi_{\mathrm{gz}}}{\psi_{\mathrm{gz}}\left(G_{1}=0\right)}=\left[\frac{G_{\mathrm{gz}}}{G_{\mathrm{gz}}\left(G_{1}=0\right)}\right]^{\mathrm{B}} \tag{5}
\end{equation*}
$$

or by substituting for $G_{\mathrm{gz}}\left(G_{1}=0\right)$ from Eq. (4) as

$$
\begin{equation*}
\frac{\psi_{\mathrm{gz}}}{\psi_{\mathrm{gz}}\left(G_{1}=0\right)}=\frac{1}{0.533^{\mathrm{B}}}\left(\frac{G_{\mathrm{gz}}^{2}}{2 g e^{3} \varrho_{\varrho} \varrho_{\mathrm{g}} d_{\mathrm{e}}}\right)^{\mathrm{B} / 2} . \tag{6}
\end{equation*}
$$

As the curve for the dependence of friction factor for the value of parameter $G_{1}=0$ is not usually available (this curve is not identical with that one for dry bed due to static holdup at $G_{1}=0$ ) and as this dependence is very close to that for dry bed and is in the cross point with the curve given by Eq. (2) very flat, instead of value $\psi_{\mathrm{gz}}\left(G_{1}=0\right)$ the value of friction factor for dry bed $\psi_{\mathrm{gs}}$ is used for Reynolds number corresponding to gas flow rate given by Eq. (4) which results in Eq. (6) mostly by change of value of the constant and only slightly in value of the exponent.

By evaluating 117 experimental data measured with a bed of spheres of 10,15 and 20 mm diameter the relation was obtained

$$
\begin{equation*}
\psi_{\mathrm{gz}} / \psi_{\mathrm{gs}}=0.48\left(G_{\mathrm{gz}}^{2} / 2 g \varrho_{\varrho} \varrho_{\mathrm{g}} d_{\mathrm{e}} e^{3}\right)^{-0.723}, \tag{7}
\end{equation*}
$$

with the mean relative quadratic deviation $15 \cdot 6 \%$ and with the correlation coefficient $r=0.9923$.

From Eq. (1) and (7) the relation for pressure drop at flooding is obtained, if for $\psi_{\mathrm{gs}}$ the friction factor for dry packed bed is substituted at the gas flow rate given by Eq. (4) and the value $z_{\mathrm{z}}$ is substituted for $z$ which may be calculated from earlier derived ${ }^{3}$ relations

$$
\begin{equation*}
z_{\mathrm{z}}=(1.177 \pm 0.0457) z_{0} \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
z_{0}=0.265\left(\frac{\sigma_{1}}{\sigma_{\mathrm{w}}}\right)^{0.1}-0.222 G_{\mathrm{gz}}^{0.25}, \tag{9}
\end{equation*}
$$

so that we obtain the final relation

$$
\begin{gather*}
\left(\Delta P_{\mathrm{g}} /\right)_{z}=0.48 \psi_{\mathrm{gs}}\left(g \varrho_{1} e^{3}\right)^{0.723}\left(2 \varrho_{\mathrm{g}} d_{\mathrm{c}}\right)^{-0.277} G_{\mathrm{gz}}^{0.554} \\
\cdot\left[e-0.312\left(\frac{\sigma_{1}}{\sigma_{\mathrm{w}}}\right)^{0.1}+0.261 G_{\mathrm{gz}}^{0.25}\right]^{-3} \tag{10}
\end{gather*}
$$

where beside physical characteristics of the packing and of fluids only the quantity $G_{g z}$ appears.

As the relation (10) holds on a high statistical level of confidence with relatively small variance, it may also be used for calculation of pressure drop for different gas flow rates in the region $G_{\mathrm{gk}} \leqq G_{\mathrm{g}} \leqq G_{\mathrm{gz}}$. For pressure drop was earlier ${ }^{3}$ derived the relation

$$
\begin{equation*}
\Delta P_{\mathrm{g}}=\psi_{1}\left(1 / d_{\mathrm{e}}\right)\left(G_{1}^{2} / 2 \varrho_{1}\right)\left(1 / z_{0}^{3}-1 / z^{3}\right) \tag{11}
\end{equation*}
$$

where $\psi_{1}$ is the factor constant for the given liquid flow rate in the whole region of gas flow rates.

For the ratio of pressure drop in an arbitrary point of the considered region to pressure drop at flooding at the same Jiquid flow rate according to Eq. (11) holds the relation

$$
\begin{equation*}
\left(\frac{\Delta P_{\mathrm{g}}}{\Delta P_{\mathrm{gz}}}\right)=\left(\frac{z_{z}}{z}\right)^{3}\left(\frac{z^{3}-z_{0}^{3}}{z_{\mathrm{z}}^{3}-z_{0}^{3}}\right) \tag{12}
\end{equation*}
$$

which by use of Eq. (8) can be transformed into

$$
\begin{equation*}
\left(\frac{\Delta P_{\mathrm{g}}}{\Delta P_{\mathrm{gz}}}\right)=2 \cdot 55\left[1-\left(z_{0} / z\right)^{3}\right] \tag{1.3}
\end{equation*}
$$

For special case of pressure drop at the beginning of liquid loading i.e. for $G_{\mathrm{g}}=$ $=G_{\mathrm{gk}}$ may be written by use of relation ${ }^{3}$

$$
\begin{equation*}
z_{\mathrm{k}}=(1.041 \pm 0.0025) z_{0} \tag{14}
\end{equation*}
$$

relation

$$
\begin{equation*}
\Delta P_{\mathrm{gk}} / \Delta P_{\mathrm{gz}}=0.283 \tag{15}
\end{equation*}
$$

The ratio of pressure drop at the gas flow rate corresponding to the beginning of li quid loading to pressure drop at flooding is constant.

In a similar way as we have derived relation (4) for maximum gas flow rate in the given packing, at which flooding takes place even at very low liquid flow rate, relation (3) may be derived for maximum liquid flow rate at which flooding of the bed takes place even at a very low gas flow rate if $G_{g z}=0$ is substituted. The following relation is then obtained

$$
\begin{equation*}
G_{1}\left(G_{\mathrm{gz}}=0\right)=0.37\left(\frac{\mu_{\mathrm{w}}}{\mu_{1}} \frac{\varrho_{\mathrm{i}}}{\varrho_{\mathrm{w}}}\right)^{0.53}\left(\frac{d_{\mathrm{e}}}{d_{\mathrm{co}}}\right)^{0.46}\left(2 g e^{3} \varrho_{1}^{2} d_{\mathrm{e}}\right)^{0.5} . \tag{16}
\end{equation*}
$$

## DISCUSSION

Obtained results are very valuable not only bécause the calculation of pressure drop in an arbitrary point of the region $G_{\mathrm{gk}} \leqq G_{\mathrm{g}} \leqq G_{\mathrm{gz}}$ is possible by use of Eqs (10) and (13) without using the friction factor $\psi_{1}$ whose calculation is affected by a greater error but also because they enable quick orientation concerning the operating region of the given packing by use of Eqs (4) and (16).

From literature it is known that mass transfer is most intensive in packed columns if hydrodynamic conditions are close to flooding. It is obvious that in practical cases the operating regime must be chosen farther from flooding with regard to reliable operation of the unit and to complexity of automatic control. But nevertheless it may be assumed that the actual operating regime will be situated in the region limited by the loading point and flooding. If for example for absorption the total amount of gas is given which should be processed and the initial and final component concentration which must be absorbed, we may determine the ratio $\left(G_{g} / G_{1}\right)$. According to the degree of control the suitable ratio $G_{\mathrm{g}} / G_{\mathrm{gz}}$ is chosen and in this way the ratio $\left(G_{\mathrm{g} z} / G_{\mathrm{l}}\right)$ is obtained which is substituted into relation (3) and for the given system and packing the liquid flow rate and the gas flow rate through a unit of cross section of packing and thus the column diameter and corresponding pressure drop per unit of the bed height may be calculated. For a complete design of the unit the mass transfer coefficients must be known. The procedure would be also the same in the case when the dimension of packing is not known subject to prior determination of this dimension from the known dependence of mass transfer coefficient and the economic analysis.

## LIST OF SYMBOLS

| A | constant in Eq. (2) |
| :---: | :---: |
| $a$ | packing surface per unit of bed volume ( $\mathrm{m}^{-1}$ ) |
| $B$ | constant in Eq. (2) |
| $e$ | porosity of dry packing |
| $d_{\text {c }}$ | characteristic length defined by Eqs (8) and (9) ${ }^{1}$ (m) |
| $d_{\text {co }}$ | characteristic length of packing taken as basis (sphere 10 mm diameter) ( 0.002797 m ) |
| $g$ | gravitational acceleration ( $\mathrm{m} \mathrm{s}^{-2}$ ) |
| G | mass flow rate of fluid per unit area ( $\mathrm{kg} \mathrm{m}^{-2} \mathrm{~s}^{-1}$ ) |
| $l$ | height of packed bed (m) |
| $\Delta P$ | pressure drop of packing ( $\mathrm{Nm}^{-2}$ ) |
| $\mathrm{Re}=G d_{\mathrm{e}} / \mu$ | Reynolds number |
| $z$ | liquid holdup per unit volume |
| $\varrho$ | density $\mathrm{kg} \mathrm{m}^{-3}$ |
| $\mu$ | viscosity $\mathrm{Nsm}^{-2}$ |
| $\psi$ | friction factor |
| $\sigma$ | surface tension $\mathrm{Nm}^{-1}$ |

Indices

| s | dry bed | k | loading |
| :--- | :--- | :--- | :--- |
| g | gas | l | liquid |
|  | o | value at $G_{g}=0$ | w |
| z | water |  |  |
|  |  |  |  |

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[^0]:    * Part VI: This Journal 35, 3344 (1970).

